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PROBABILITY AND PROOF IN *STATE v. SKIPPER*: AN INTERNET EXCHANGE

Ronald J. Allen, David J. Balding, Peter Donnelly,
Richard Friedman, David H. Kaye, Lewis Henry LaRue
Roger C. Park, Bernard Robertson, Alexander Stein*

This is not a conventional article. It is an edited version of messages sent to an Internet discussion list.¹ The listings begin with the mention of a recent opinion of the Connecticut Supreme Court, parts of which are reproduced below. The listings soon move to broader issues concerning probability and other formal systems, their limitations, and their uses either in court or as devices for understanding legal proof.²

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1. This list, bayesian-evidence@massey.ac.nz, is maintained by Bernard Robertson of the Massey University, New Zealand. Additions and deletions have been made to the original messages to enhance the clarity of the remarks. For the sake of brevity and focus, many messages distributed on the list have not been included. The material in the footnotes was prepared by the editor, David H. Kaye, with contributions from Richard Friedman.

2. Many of these broader issues have been discussed in other publications. *See generally*, e.g., *PROBABILITY AND INFERENCE IN THE LAW OF EVIDENCE: THE LIMITS AND USES OF BAYESIANISM* (Peter Tillers & Eric D. Green eds., 1988); Symposium, *Decision and Inference in Litigation*, 13 *CARDOZO L. REV.* 253-1079 (1991). To avoid excessive repetition, some of

State v. Skipper

228 Conn. 610, 637 A.2d 1101 (1994)³

CALLAHAN, ASSOCIATE JUSTICE.

The dispositive issue in this appeal is the admissibility of the probability of paternity statistic calculated from DNA evidence. The defendant was [convicted of multiple counts of various crimes involving sexual conduct with a minor but acquitted of simple assault.] The trial court sentenced the defendant to a term of imprisonment of 24 years, execution suspended after 12 years, followed by five years probation. Thereafter, the defendant appealed to the Appellate Court. We transferred his appeal to this court We reverse the judgment of the trial court.

The jury could reasonably have found the following facts. The defendant began to make sexual overtures to the victim, who was the daughter of a neighbor and a friend of his own daughter, sometime in 1982, when the victim was approximately eight years old and in the third grade. In response to the victim's protests, the defendant told her that he would leave her alone if she allowed him to take topless photographs of her. When the victim complied, however, the defendant persisted in his contact with her, threatening to show the photographs to her friends if she refused to see him. By the time the victim was in

the fourth grade, the defendant had begun to molest her physically. At some point, while she was still in the fourth or fifth grade, the defendant began to have sexual intercourse with the victim. Even after she moved out of his neighborhood, the defendant continued on a regular basis to have sexual relations with the victim in his van at various locations.

When the victim was in the tenth grade, she told the defendant that she wanted him to leave her alone and that she no longer cared what he did with photographs he had taken. At that time, the victim's parents were in the process of attempting to adopt a little girl. When the victim persisted in her refusal to see him, the defendant intimated to her that the adoption would not go through if the authorities were notified of their relationship.

The sexual relationship between the defendant and the victim continued until March, 1989. At that time, the defendant, in an attempt to end a platonic friendship between the victim and a classmate named Marvin, told her to advise Marvin that she was pregnant and provided her with a falsified home pregnancy test that ostensibly displayed a positive result. Marvin promptly informed the victim's parents. When

the messages discussing these matters have been truncated, and others have been omitted.

3. Some footnotes and citations are omitted without further notice. The footnote numbers are those in the original opinion.

confronted by her mother, the victim broke down and told her of the nature of her relationship with the defendant.

On March 9, 1989, the victim gave a statement to the police. The next day, the victim's mother took her for a medical examination that revealed that the victim was in fact pregnant. On March 22, 1989, the victim had an abortion. . . .

The defendant claims that the trial court improperly admitted testimony of the probability of paternity percentage based on DNA testing. We agree and, on this basis, reverse the judgment of the trial court and remand the case for a new trial.

Kevin McElfresh, the state's expert witness and the director of Identity Testing Laboratories of Lifecodes Corporation (Lifecodes), testified at trial regarding the defendant's paternity index. The paternity index is an odds ratio, based on DNA tests, measuring the likelihood that the defendant would produce a child with the same phenotypes as the fetus in question as compared to an unrelated random male. The paternity index in this case was 3496, indicating that only one out of 3497 randomly selected males would have the phenotypes compatible with the fetus in question.⁹

9. Although the defendant asserts in his brief that there are 340 other individuals in Bridgeport who could have been the biological father based on the paternity index, we are unable to determine from the record how he arrived at that number. Our mathematical calculations, arrived at by dividing the number of African-American males, according to

McElfresh further testified that the paternity index could be converted into a statistic indicating the percentage of the defendant's probability of paternity. In the present case, he testified that he had made that conversion and that the percentage of probability that it was the defendant who had fathered the fetus was 99.97%. The usual method for calculating the probability of paternity, and the method that McElfresh used in the present case, is Bayes' Theorem. . . . In the context of determining paternity, Bayes' Theorem postulates the multiplication of the paternity index, i.e., the new statistical evidence, by an assumed prior percentage of probability of paternity in order to obtain a new percentage of probability of paternity. In order to assume a prior

the 1990 census, in Connecticut and the United States by the paternity index, indicate that there are approximately 20 African-American males between 16 and 50 years old in Connecticut, and 2138 in the United States, who could have been the biological father of the victim's fetus. The probability of paternity percentage cannot, however, tell us which of these males is the father.

Editor's note: The court's characterization of the paternity index as an "odds ratio" and its interpretation of this quantity as the rate at which the tests would include men as genetically possible fathers are incorrect. See, e.g., Mikel Aickin & David Kaye, *Some Mathematical and Legal Considerations in Using Serological Tests to Prove Paternity, in INCLUSION PROBABILITIES IN PARENTAGE TESTING* 155 (R.H. Walker ed., 1983); C.C. Li & A. Chakravarti, *An Expository Review of Two Methods for Calculating the Paternity Probability*, 43 AM. J. HUM. GENETICS 197 (1988).

probability of paternity, however, it is also necessary to assume a prior probability of intercourse.

In Bayes' Theorem, the prior probability of paternity is not cast as any particular figure. Generally, experts who testify in paternity proceedings choose a number to represent the prior probability. Most experts, as did McElfresh here, set the prior probability at 50%, expressed as odds of one, i.e., 50-50, reasoning that 50% is a neutral starting point because it assumes that it is just as likely that the defendant is not the father as it is that he is the father.

Our criminal justice system is built upon the premise that the prosecution must prove "every fact necessary to constitute the crime with which [the defendant] is charged' beyond a reasonable doubt." *State v. Salz*, 226 Conn. 20, 28, 627 A.2d 862 (1993), quoting *In re Winship*, 397 U.S. 358, 364 (1970). The right to have one's guilt proven beyond a reasonable doubt is of constitutional dimension. In a sexual assault prosecution, sexual intercourse is an element that must be proven by the state beyond a reasonable doubt. The utilization of Bayes' Theorem by the prosecution, however, permitted the introduction of evidence predicated on an assumption that there was a 50-50 chance that sexual intercourse had occurred in order to prove that sexual intercourse had in fact occurred.¹⁰ The 50-50 assumption that

sexual intercourse had occurred was not predicated on the evidence in the case but was simply an assumption made by the expert. . . .

The assumption that there is a substantial possibility that the defendant had intercourse with the victim, however, raises serious concerns in sexual assault cases. It is antithetical to our criminal justice system to presume anything but innocence at the outset of a trial. It is not until the defendant has been convicted that the presumption of innocence disappears. "The defendant's presumption of innocence until proven guilty is an 'axiomatic and elementary' principle whose 'enforcement lies at the foundation of the administration of our criminal law.'" *State v. Allen*, 205 Conn. 370, 376, 533 A.2d 559 (1987), quoting *Coffin v. United States*, 156 U.S. 432, 453 (1895). The presumption allocates the burden of proof in a criminal trial to the state. "[T]o implement that presumption, 'courts must be alert to factors that may undermine the fairness of the fact finding process. In the administration of criminal justice, courts must carefully guard against dilution of the princi-

ity in paternity cases relates to paternity, not intercourse. Since intercourse ordinarily is a prerequisite to paternity, however, the prior probability of paternity provides a lower bound for the prior probability of intercourse. If the probability that a given man is a father is one-half, and if the child was conceived as a result of intercourse rather than in vitro fertilization or some other procedure, then it cannot be any less probable that the man had intercourse with the mother; but it can be more probable that he did.

10. *Editor's note:* The prior probabilit-

ple that guilt is to be established by probative evidence and beyond a reasonable doubt. In re Winship, [*supra*, 397 U.S. at 364].”

Without first assuming a prior probability of paternity, i.e., guilt, Bayes’ Theorem cannot be applied, and the probability of paternity cannot be computed in sexual assault cases. Because Bayes’ Theorem requires the assumption of a prior probability of paternity . . . , its use is inconsistent with the presumption of innocence in a criminal case such as this, in which Bayes’ Theorem was used to establish the probability of paternity, i.e., that the defendant was the father of the product of conception of an alleged sexual assault. Whether a prior probability of 50% is automatically used or whether the jury is instructed to adopt its own prior probability,¹⁸

18. It has been suggested that jurors be shown a chart illustrating a range of prior probabilities and the resulting probabilities of paternity. Permitting the jury to derive its own prior probability, however, still implicates the presumption of innocence. See, e.g., L. Tribe, *Trial by Mathematics: Precision and Ritual in the Legal Process*, 84 HARV. L. REV. 1329, 1368-75 (1971):

It may be supposed that no juror would be permitted to announce publicly in mid-trial that the defendant was already burdened with, say, a 60% probability of guilt—but even without such a public statement it would be exceedingly difficult for the accused, for the prosecution, and ultimately for the community, to avoid the explicit recognition that, having been forced to focus on the question, the rational juror could hardly avoid reaching some such

answer. And, once that recognition had become a general one, our society’s traditional affirmation of the ‘presumption of innocence’ could lose much of its value.

Id. at 1370. Moreover, allowing the jury to adopt a prior probability and, hence, arrive at a probability of guilt, raises concerns in criminal cases regarding the burden of proof of guilt beyond a reasonable doubt. In adopting a prior probability of guilt and viewing the corresponding probability of paternity on a chart, the jury is left “with a number that purports to represent [its] assessment of the probability that the defendant is guilty as charged. Needless to say, that number will never quite equal 1.0, so the result will be to produce a quantity . . . which openly signifies a measurable . . . margin of doubt” *Id.* at 1372; see also R. Jonakait, *When Blood Is Their Argument: Probabilities in Criminal Cases, Genetic Markers, and, Once Again, Bayes’ Theorem*, 1983 U. ILL. L. REV. 369, 415-20 (1983). “[A]ny conceptualization of reasonable doubt in probabilistic form is inconsistent with the functional role the concept is designed to play.” C. Nesson, *Reasonable Doubt and Permissive Inferences: The Value of Complexity*, 92 HARV. L. REV. 1187, 1225 (1979); see also *State v. DelVecchio*, 191 Conn. 412, 417-18, 464 A.2d 813 (1983) (jury instruction using a football field simile and instructing the jury that “it . . . is up to you to decide” where reasonable doubt lies between the 50 yard line and one hundred yard line diluted the constitutional standard of proof beyond a reasonable doubt). Allowing jurors to reach their own prior probability also presents practical problems. See L. Tribe, 84 HARV. L. REV., *supra*, at 1359-66. We cannot say that merely introducing a chart illustrating a range of prior probabilities for educational purposes without requiring the jury to adopt a specific figure would alleviate our concerns. “[W]hether the benefits of using this method of statistical inference solely to educate the jury by displaying the probative force of the evidentiary findings

when the probability of paternity statistic is introduced, an assumption is required to be made by the jury before it has heard all of the evidence—that there is a quantifiable probability that the defendant committed the crime. In fact, if the presumption of innocence were factored into Bayes' Theorem, the probability of paternity statistic would be useless. If we assume that the presumption of innocence standard would require the prior probability of guilt to be zero, the probability of paternity in a criminal case would always be zero because Bayes' Theorem requires the paternity index to be multiplied by a positive prior probability in order to have any utility. "In other words, Bayes' Theorem can only work if the presumption of innocence disappears from consideration."

We conclude that the trial court should not have admitted the expert

testimony stating a probability of paternity statistic. Moreover, we cannot say with any degree of confidence that a probability of paternity statistic of 99.97%, as testified to by the state's expert, would not have influenced the jury's decision to convict the defendant of both sexual assault and risk of injury. Because the admissibility of the probability of paternity statistic involves a constitutional issue, and because we cannot say that the admission of that statistic here was harmless beyond a reasonable doubt, a new trial is required. . . .

would be worth the costs in terms of time-consumption and possible confusion is a close . . . question." 1 C. MCCORMICK, EVIDENCE (4th ed. 1992) § 210, at 959; D. Kaye, *The Admissibility of 'Probability Evidence' in Criminal Trials—Part II*, 27 JURIMETRICS J. 160, 171 (1987); but see *State v. Spann*, 130 N.J. 484, 518, 617 A.2d 247 (1993) ("[w]e ... believe that appropriate jury instructions can cure all of [the problems with letting the jury derive its own prior probability], or at least diminish their risk to the point that the advantages of the expert's calculation outweigh these risks, assuming the opinion is otherwise admissible"). For an exhaustive explanation of the practical problems associated with a chart approach, see L. Tribe, 84 HARV. L. REV., *supra*, at 1358-68, and R. Jonakait, 1983 U. ILL. L. REV., *supra*, at 403-405.

Date: 29 July 1994

From: Bernard.Robertson@Hertford.oxford.ac.uk

Initial reactions to *State v Skipper*:

1. It is not the job of a witness to assume prior probabilities, but only to testify as to the strength of the evidence.

2. Therefore, the witness should not apply Bayes's Theorem by multiplying the likelihood ratio by any prior, but should simply testify as to the likelihood ratio. In the context of paternity, this has been the subject of so much uncontested writing that it is staggering how it has not penetrated the professional consciousness at all.

3. All that the court required to find to reject the evidence was that the witness should not assume or conceal a prior. This finding is welcomed. The remainder of that section of the judgment is obiter dictum.

4. Any comment capable of being interpreted as meaning that Bayes's Theorem should not be applied by the jury is not required for the decision, is not justified by the reasoning, and is wrong.

5. The presumption of innocence does not require assessing a prior probability of zero for the following reasons:

a. A prior probability of zero is simply a formal way of saying "I have an unshakable belief, which cannot be affected by any evidence, that chummy did not do it." Any juror who announced this would presumably be disqualified.

b. The presumption of innocence is simply a restatement of the burden and standard of proof. The Supreme Court of Connecticut has here fallen for a persistent fallacy that should have been laid to rest by now. *See* 9 WIGMORE ¶ 2511.

The key part of this is explained perfectly well in D.H. Kaye, *The Probability of an Ultimate Issue: The Strange Cases of Paternity Testing*, 75 IOWA L. REV. 75 (1989), which is quoted from by the court on one narrow point but the message of which has not apparently been understood.

Date: 30 July 1994

From: Alexander Stein <msalex@pluto.mscc.huji.ac.il>

I agree entirely with Bernard's first point. As for the second, the question is how to instruct the jury about the use of the likelihood ratio; this should entail a direction about the prior odds. The reading of judgment in Bernard's third numbered paragraph is not altogether unbiased. In any event, the ratio-obiter distinction is passé. The fourth point is right only if the fifth is correct. I agree with 5(a), not with 5(b): the presumption of innocence has a broader significance.

But this is enough to show that Bernard's critique of *Skipper* is essentially justified. Yet, if the jury were instructed: "You have to determine

the complainant's credibility in light of the DNA evidence that has shown that only one out of 3,497 males has the same genetic traits as that common to the defendant and the fetus," would there be any reasonable doubt that the complainant might have picked *Skipper* at random to falsely accuse him of molesting her? To answer this question, one does not have to be a statistician; no mathematics is required here, just common sense—and here lies the problem for imperialistically inclined Bayesians.

Date: 30 July 1994

From: Richard.Friedman@um.cc.umich.edu

I never know what "imperialistic Bayesianism" is supposed to mean. Those of us who use Bayesian probability as an analytical tool (and who regard as necessarily problematic groupings of subjective beliefs that are not consistent with each other according to the standard principles of probability theory) do not necessarily suggest that the jury should be invited to apply Bayesian statistics; in most cases, it would be foolish even to think of doing so.

But when the key evidence itself is statistical, I wonder. I'm not so sure juries will use their common sense correctly—Kahneman and Tversky suggest they may use it very poorly.⁵ They may, for example, say, "That means the probability of innocence is only 1/3497," or at the other extreme, "Well, if one in 3497 people have this genetic type, that must mean there are thousands in the population who do, and that means the chance is only one in several thousand that this suspect is the right person." So then I wonder, should the jury be given a little help, being told not only what the likelihood ratio is, but what the significance of the likelihood ratio is, i.e., how to combine it with whatever probability the jury might have assessed apart from this evidence? Do I understand Bernard to say that this would be inappropriate? (Notice that the expert giving this testimony (or the judge by instruction?) wouldn't have to be a DNA expert—just somebody who can explain the necessary principles of probability.)

Also, instead of testifying as to the likelihood ratio only, should the expert be allowed to testify as to the components—i.e., probability of the evidence given the truth of the hypothesis and probability of the evidence given the falsity of hypothesis?

5. See, e.g., D. KAHNEMAN, P. SLOVIC & A. TVERSKY, JUDGMENT UNDER UNCERTAINTY: HEURISTICS AND BIASES (1982).

Date: 30 July 1994

From: Alexander Stein <msalex@pluto.mscc.huji.ac.il>

"Imperialistic Bayesianism" refers to a theory that attempts to exclude a Baconian type of reasoning from the domain of rationality. One may reason about uncertainties by assessing the strength of the evidence which supports or weakens one's hypothesis, etc., as Jonathan Cohen explains.⁶ Calculation of chances which entails application of the Bayes's theorem is another framework of reasoning. Kahneman and Tversky have imperialistically postulated that rationality has got to be Bayesian and then shown that laymen are consistently fallacious in reasoning under uncertainty. I am sorry for making myself Cohen's gramophone again, but: whose is the fallacy, if there is another rational, namely Baconian, method of handling uncertainties?⁷

Richard also asks whether "the jury should be given a little help, being told not only what the likelihood ratio is, but what the significance of the likelihood ratio is, i.e., how to combine it with whatever probability the jury might have assessed apart from this evidence." Once it is recognized that the beyond-a-reasonable-doubt standard is not just a question of fact, the judge would be better advised to leave the jury alone with the likelihood ratio and the rest of the evidence.

If the jury in *Skipper* had been instructed by focusing on the credibility question—Could the complainant have picked Skipper at random, given the remarkably high paternity index? Skipper would have already been in prison. (He will be there anyway, after his new trial). My students here have been appalled by the judgment, and I think that the desire to be scientific rather than intuitive is at least partly responsible for what happened in this case.

Date: 30 July 1994

From: Roger Park <parkx003@maroon.tc.umn.edu>

Footnote 18, rejecting the Bayesian chart method of presentation, may technically be *obiter*, in the sense of not being necessary for the decision, but it is certainly not a casual statement. A lower court judge in Connecticut would have to be pretty contemptuous to allow a chart with prior and posterior probabilities to be presented after *Skipper*.

I think what the expert did in *Skipper* is pretty outrageous (testifying to a probability of paternity without even revealing that it is based upon an assumed prior of .5). So it's not surprising that the Connecticut Supreme Court reached the result it did, and it would be supremely easy to reach that

6. See L. JONATHAN COHEN, *THE PROBABLE AND THE PROVABLE* (1977).

7. L. Jonathan Cohen, *On the Psychology of Prediction: Whose Is the Fallacy?* 7 COGNITION 385 (1979).

result without rejecting any form of Bayesianism. But the court went out of its way to reject the chart presentation, and, citing Tribe and Nesson, to lay out its theory for doing so. Maybe it will change its mind when presented with the issue in another form, but I doubt if it will be presented with the precise issue discussed in footnote 18 because I don't think prosecutors will try it after that footnote.

Maybe I am simplistic, but I wonder if a Bayesian presentation is more trouble than it's worth when there is a lot of nonstatistical evidence against the defendant and the expert is ready to testify that the population frequency is 1/3497. Subjects may fall into the fallacies that Rich describes in paper experiments, but any prosecutor or defense counsel who can't explain in argument why those ways of looking at the evidence don't make sense ought to look for some other job. Or for that matter, the expert who is ready to explain Bayesian analysis could explain how to avoid the fallacies, an approach that would not require the jury to formulate a prior or anything like that.

Date: 1 August 1994

From: Richard.Friedman@um.cc.umich.edu

Maybe Roger is right, but my problem is, I'm not sure how you would explain why they're fallacies, or how best to use the evidence, without introducing some notion of a prior probability—because after all that *is* the way evidence of this type has to be used to be used correctly. Oops! Am I revealing imperialism again? Let's just say that the probability that the jury assesses after hearing the statistical evidence should be consistent with that evidence and with the probability it would assess without that evidence according to the laws of probability, or else there's a problem.

So Roger, here's a question—What *would* you say if you were a prosecutor (or a defense attorney) that would help the jury use the evidence properly, and yet avoid any use of a prior? Or is it not the abstract use of a prior—i.e., “you have to combine this evidence with all of the other evidence in the case, and the more probable guilt seemed before, the more probable it should seem afterwards”—but the attempt to make this more precise, or even quantify it, that's objectionable?

Date: 5 August 1994

From: Roger Park <parkx003@maroon.tc.umn.edu>

Well, admittedly it may not be as easy for me to do it as to say it can be done, but here are my off-the-cuff thoughts about how to help the jury

avoid the "prosecutor's fallacy" or the "defense counsel's fallacy"⁸ without doing a full Bayesian chart presentation.⁹

(1) Prosecutor's fallacy (that the 1/200 population frequency of the incriminating trait means that there is a 1/200 chance that the defendant who possesses it is innocent). This can be combatted just by presenting the non-fallacious part of the defense counsel's fallacy. Ask the prosecution expert "Isn't it true that there are approximately 10,000 other men in the Los Angeles area who have those same blood characteristics?" Have one of the experts explain that a 1/200 random match probability does not mean that there is a 1/200 probability that someone other than the defendant was the source.

(2) Defense counsel's fallacy (that the fact that there are 10,000 other men in the area with the same trait means that the odds are 10,000:1 that the defendant is innocent). Intuitively, it seems to me that this fallacy is just not all that potent in its power to persuade. (I hope I am not overlooking some large body of research that shows me wrong—it seems to me that the Thompson and Schumann article, good as it is, is not conclusive on this point.)¹⁰ In most contexts it's such a laughable argument you wouldn't expect it to be made. In a case in which the defendant is charged with robbery and tries to explain his possession of the stolen property by saying he won it in a poker game, would the defense counsel try to argue "the victim recognized him by his tattoo, and there are 10,000 other men in Los Angeles who have that type of tattoo, therefore the chance of his guilt is one in 10,000"? The argument is almost as laughable in the forensic science context and possibly could be answered by something like "Defense counsel would have you believe that at most there is a 1/10,000 chance that the defendant is guilty because there are 10,000 other people who share the defendant's blood type. He would have you believe that just by coincidence the victim identified the defendant as her attacker, and just by coincidence he later turned out to have the same type of blood found under her fingernail, blood that 99.5% of the population doesn't have. He would have you believe that when you put those two pieces of evidence together you get a 1/10,000 chance of guilt. All I ask is, use your common sense. When you put all the evidence together, it is

8. See William C. Thompson & Edward L. Schumann, *Interpretation of Statistical Evidence in Criminal Trials: The Prosecutors' Fallacy and the Defense Attorney's Fallacy*, 11 LAW & HUM. BEHAV. 167 (1987). See also David J. Balding & Peter Donnelly, *The Prosecutor's Fallacy and DNA Evidence*, [1994] CRIM. L. REV. 711.

9. See Ira Ellman & David Kaye, *Probabilities and Proof: Can HLA and Blood Group Testing Prove Paternity?* 54 NYU L. REV. 1131 (1979) (describing the "chart method").

10. For reviews of studies of probability estimates by mock jurors, see D.H. Kaye & J. Koehler, *Can Jurors Understand Probabilistic Evidence?* 154 J. ROYAL STAT. SOC'Y (A) 751 (1991); William C. Thompson, *Are Juries Competent to Evaluate Statistical Evidence?* 52 LAW & CONTEMP. PROBS. 9 (1989).

overwhelming evidence of guilt.” I suppose one could also add some idea that if the jury thought the evidence was strong before, after the blood test is added it only gets stronger, not weaker.

I am sure there are people who could do it better than that. (I tried to find out by asking our local DNA prosecutor this morning, but he said “it doesn’t come up.”)

I hope I didn’t give the impression that I was totally against the use of Bayesian concepts in argument or expert testimony. I’m not against anything that smacks of the “notion of a prior.” All I meant to suggest was that, when the population frequency is small and there is significant other evidence against the defendant, a Bayesian chart presentation may be unnecessary and perhaps more likely to mislead than to help. Formulating a mathematical prior probably is an unfamiliar task for jurors that may be misunderstood by some of them. Also, the expert might present the chart in a way that causes jurors to overlook the danger of laboratory error or the danger that the forensic expert was influenced by what he or she heard about the strength of the rest of the case, or to overlook “soft variables” such as an innocent explanation for the presence of the defendant’s trace at the scene. A presentation that tried to allow the jurors to take those dangers into account in converting a numerical prior to a numerical posterior would, I believe, get rather complicated.

Date: 5 August 1994

From: Bernard.Robertson@Hertford.oxford.ac.uk

1. This assumption of a prior of .5 may be logically outrageous but it has been the standard way of giving paternity evidence for decades. Furthermore, in the United States and in some jurisdictions such as Germany, it is usual to talk about a probability of contact, or presence, using the same formula.

All the great statements from judges about “mathematical probability” not being the same as legal probability come from cases where an expert has given evidence in this form. This is just another on the list.

2. The answer to Alex of course is that we are not talking about life or nature but how to think. There can only be one logical way to think. For a recent demonstration of that logic requires the axioms, see C.R. Smith & G. Erickson, *From Rationality and Consistency to Bayesian Probability*, in MAXIMUM ENTROPY AND BAYESIAN METHODS 29-44 (J. Skilling, ed., 1989).

Date: 4 August 94

From: Peter Donnelly <P.J.Donnelly@qmw.ac.uk>

I want to comment on general issues raised by the recent email discussion prompted by the *Skipper* decision. I will use Bayesian ideas in the sense Richard described as “an analytical tool,” without suggesting that they should necessarily be inflicted on the jury.

1. Richard asked “instead of testifying as to the likelihood ratio only, should the expert be allowed to testify as to the components—i.e., probability of the evidence given the truth of the hypothesis and probability of the evidence given the falsity of hypothesis?” Presumably the expert would be required to give these components separately if asked, and for example defence counsel would have the right to so ask. If not, how else can the expert be cross-examined on aspects of the numerical value of the likelihood ratio that is presented. (This seems particularly apposite in settings in which different experts may give different likelihood ratios, or in which some assumptions made by the expert in calculating a likelihood ratio are not universally accepted.)

2. Roger wrote “Subjects may fall into the fallacies that Rich describes in paper experiments, but any prosecutor or defense counsel who can’t explain in argument why those ways of looking at the evidence don’t make sense ought to look for some other job.”

I am hesitant in addressing legal matters as these are beyond my expertise, and well within that of all the other contributors to the discussion. Nonetheless, can it be that “any good lawyer would point out such and such to the jury” is a sensible principle on which to base procedure? Would all other rules of evidence, for example, survive in the presence of this principle? Not all defendants have good lawyers.

Secondly, the errors to which reference was made, the so-called prosecutor’s fallacy and defence attorney’s fallacy, are errors of logic, or in reasoning with probabilities. My experience in the United Kingdom is that otherwise extremely distinguished lawyers will not immediately recognize these as errors. They are quick to accept the point when it has been explained. In other words, I don’t agree that even good lawyers (who haven’t thought about the issues previously) will necessarily see the fallacies.

3. Alex commented “Kahneman and Tversky have imperialistically postulated that rationality has got to be Bayesian and then shown that laymen are consistently fallacious in reasoning under uncertainty . . . but whose is the fallacy, if there is another rational, namely, Baconian, method of handling uncertainties?”

Even if one accepts another form of rationality, it does not seem to follow that the “errors” of reasoning (within the coherent Bayesian framework) to which Richard refers (namely prosecutor’s and defence

attorney's fallacy) are correct in the other framework. Is Alex saying that these particular arguments *are* correct in his framework?

My own view is that there is a very real danger of a jury hearing a frequency of one in a million and erroneously thinking that this is the chance that the defendant is innocent (or of interpreting a likelihood ratio of one million as meaning that it is a million times more likely that the defendant is guilty than innocent) and going from this incorrect understanding of the evidence to a conviction.¹¹ This reasoning, about which I would be concerned, is different from saying that there is another rational way of assessing the likelihood ratio, without a prior, on the basis of which the jury reach a particular conclusion.

4. In general there will be more than one relevant likelihood ratio. (I will phrase the discussion in terms of DNA evidence in a paternity case, but it applies to criminal cases and to evidence other than DNA.) One way of seeing this is to observe that what Richard describes as "the falsity of the hypothesis [of guilt]" is a disjunction. The probability of the evidence will be different (in general) for different alternative sources of the DNA. An extreme example would be the difference between the probability of the DNA evidence if the true father were a brother of the alleged father and the probability if the true father were an unrelated individual from a different racial group. Between these two extremes lie many others, depending on the degree of relationship between true and alleged father (where relationship here may refer simply to members of particular population subgroups).

If one is to be Bayesian, there is a single correct way of assessing the DNA evidence. It involves combining the different likelihood ratios in a way which depends on one's prior beliefs (here prior actually means "after the other evidence") about who the true father might be. (That is, in a Bayesian formulation, not only does the assessment depend on the prior probability of the defendant's guilt, *it also depends on the prior probability of guilt for all other possible culprits.*)

I make this point for two reasons. The first is that it seems to me to be important—many so-called Bayesian proscriptions for assessing evidence appear not to have appreciated (a) that there are different likelihood ratios and (b) that they must be combined in a certain way which depends on prior beliefs. Secondly, the "correct" Bayesian solution is thus more complicated than might have been supposed. This seems to me to impinge on the discussion of the extent to which the use of likelihood ratios should be explained to juries (either by judges or the experts). It is one thing to say that an expert should give a likelihood ratio. In fact, they should be providing lots of likelihood ratios—which of these actually matter will

11. David J. Balding & Peter Donnelly, *Inference in Forensic Identification* 159 J. ROY. STAT. SOC'Y A 21 (1995).

depend on the jury's assessment of the other evidence. If one is to be Bayesian, it seems more difficult to argue that these likelihood ratios should simply be presented to the jury with no further comment.

As a slight aside, I have pointed out on this network before that there are various relevant likelihood ratios. My impression is that others disagree, arguing that by the end of the trial both sides will have settled on particular (classes of) alternatives, thus reducing the "not guilty" hypothesis to a class of alternative culprits for whom there is a single probability (of the evidence for this class of culprit) to feed into the likelihood ratio calculation. I cannot accept this argument.

(i) It amounts to saying that the lawyers involved have the right to say to the jury at the end of the trial that they can only consider certain possible alternatives. This is equivalent to them being instructed to assign zero or negligible prior to certain individuals as possible culprits. You can't be half Bayesian. If you believe it is the jury's job to assess priors, then it is their job, not the lawyers'.

(ii) As an empirical fact, I don't believe all cases will fit this description. It is perfectly possible in a paternity case that (at the end of the trial) both the brother of the alleged father, and an unspecified unrelated individual, will be plausible true fathers. Neither the prosecution, defence, nor jury should be forced to accept only one of these alternatives. (Of course it may be in the defence's best interests to argue that it was the defendant's brother, but this is another matter.)

Date: 6 August 1994

From: Ron Allen <RJALAF@nuls.law.nwu.edu>

Peter Donnelly's remarks seem to me exactly right and express in a very succinct fashion why Bayes's theorem can only be a small part of the action, so far as the nature of juridical proof is concerned. Note, I don't say "no part of the action." There are certainly cases where application of Bayes theorem is useful, and maybe even helpful, but it explains virtually nothing. Take any murder case. There is a positive even if quite small probability that anybody with the opportunity committed that crime. Prior to investigation, in a city like Chicago, that means over 5,000,000 people. If the Bayesian imperialists' account of juridical proof were true, the prosecution would have to investigate all of those possibilities and demonstrate their implausibility. Without doing so, one cannot possibly say anything interesting, in a Bayesian sense, about the defendant. Yet, we know that is not done. O.J. Simpson will not be acquitted by the defense pointing to the Los Angeles phone book and saying "But any one of these people could have done it, and 5,000,000 low probability cases must sum to reasonable doubt." Yet, if Bernard's arguments were accurate, at least if, as Peter points out, he took his own arguments seriously, O.J. would get off. He won't because the nature of

juridical proof is not well explained by Bayes's Theorem. Other forms of reasoning, which as most of you know I've tried to discuss in my work over the last few years and will not repeat here, are at play. Only after the case is almost over is there typically even a question that can be formulated usefully in a Bayesian fashion, and it is only at that stage that Bernard's arguments have any force. But, to extrapolate from this unusual occurrence to the nature of the process at a whole is to mistake the unusual for the typical.

Note that I refer to Bernard's arguments, and not to Rich Friedman's. That is because more and more Rich is simply using the term "Bayesianism" to refer to reason. Bernard, by contrast, continues to use the term in its conventional sense, which refers to one highly specified manner of reasoning. That manner of reasoning has formal requirements, which Bernard continually and without exception ignores in making what Alex rightly calls his imperialistic claims, as Peter's comments accurately point out.

Date: 8 August 1994

From: DH Kaye <k@asu.edu> and David J. Balding

David Balding and I have some thoughts to add to the flurry of notes on *State v. Skipper*. We see many problems with the opinion.

1

Some of the problems are technical. The court mischaracterizes the paternity index as an odds ratio, and it then misinterprets the index as a likelihood ratio with regard to inclusion of a tested man rather than with regard to paternity. In note 11, the court confuses the paternity index with the "test evidence" that is involved in computing the index, and it equates the prior probability of paternity to the prior probability of intercourse. None of these make much difference to the outcome or the reasoning in the case.

2

a. The court seems to rely on two constitutional principles for holding that the use of an undisclosed prior probability of one-half is error. First, it relies on the "right to have one's guilt proven beyond a reasonable doubt." The use of a prior of one-half offends this principle, as far as we can tell, because it gives the prosecution a head-start in proving its case. If one conceives of the DNA evidence coming after other evidence in the case, then the "prior" probability reflects that evidence and may be lower than one-half. If one conceives of the DNA evidence as the first datum, then some prior probability has to be assumed. We discuss this in connection with the "presumption of innocence" below, where we suggest that the prior should be closer to zero than to one-half. The point here is that the burden of

persuasion concerns the probability required for a conviction and does not preclude the updating of probabilities via Bayes' theorem. In notes fifteen and eighteen, the court disavows the idea that a juror should adopt explicit probabilities of intercourse, but it is difficult to see how this result follows from the burden of persuasion. The court quotes Tribe for the view that any posterior probability less than one indicates "measurable doubt" and therefore should not be acknowledged. Why the requirement of proof beyond a reasonable doubt bars some measurable doubt is left unstated.¹²

b. The second basis for the holding has to do with the "presumption of innocence." The court writes that the presumption requires the use of a prior probability of zero, which always would yield a posterior probability of zero. This is not a criticism of Bayes's theorem, either as an analytical device or as an explicit tool at trial, but an obvious misconception. As Bernard points out, a prior probability of zero is not a presumption of innocence, but rather an unalterable (and impermissible) conclusion of innocence. In a Bayesian framework, the presumption can be interpreted in two ways. Bernard gives one view, which is that the presumption is just another way of saying that the prosecution must adduce enough evidence to dispel all reasonable doubt. Roughly speaking, another view is that the presumption means that before evidence is presented, the defendant should be thought no more likely than anyone else to be guilty.¹³ This translates into a small, but non-zero prior probability. Since a prior probability of one-half (at the outset) violates this conception of the presumption of innocence, the court's conclusion is consistent with a Bayesian analysis.

3

We share Bernard's disappointment over the continued perversion of Bayes's theorem in paternity cases. More sensible applications of the theorem to the paternity problem have been described, and the American Association of Blood Banks' Standards on Parentage Testing require an expert at least to disclose the prior probability. Are there other rational ways to present the evidence in cases like *Skipper*? Of course. One can argue, as this court does, that the benefit of presenting explicit calculations of a posterior probability (or probabilities) to jurors might be outweighed by the disadvantages. The issue seems to turn on questions of juror psychology, as Richard Friedman, Roger Park and Peter Donnelly note.

12. See generally Daniel Shaviro, *Statistical-Probability Evidence and the Appearance of Justice*, 103 HARV. L. REV. 530 (1989).

13. See, e.g., A.P. Dawid, *The Island Problem: Coherent Use of Identification Evidence*, in ASPECTS OF UNCERTAINTY 159, 169 (P.R. Freeman & A.F.M. Smith eds., 1994) ("the presumption of innocence is fully in accord with Bayesian reasoning: before any evidence is adduced, we should treat the accused as exchangeable with all other members of the population").

Are there other rational ways to reason about the evidence? This depends on what one means by "rational."¹⁴ Just as alternative systems of deductive logic have been proposed, competing versions of inductive logic have been devised. As Alex points out, Jonathan Cohen's "Baconian" system is one. The appeal of using conventional probability theory to describe "rational" degrees of belief is that any other system leads to inconsistencies in the degrees of belief assigned to logically equivalent propositions.

This is not to say that Bayesian treatments are necessarily simple. Much of the legal literature confines itself to two hypotheses, like guilt and innocence. As Peter notes, the hypothesis of innocence is a composite hypothesis. Ron's position that composite hypotheses are intractable seems unconvincing. Ron seems to think that if there are five million people in a city, then "the Bayesian imperialists' account of juridical proof" implies that the prosecution would have to obtain specific, falsifying evidence with regard to the guilt of 4,999,999 people, and that without such evidence, the many "low probability cases must sum to reasonable doubt." But evidence that increases the posterior probability of one hypothesis necessarily decreases the sum of the probabilities of the remaining hypotheses, and if the prosecution's case against one defendant (O.J. Simpson, to use Ron's example?) is strong enough, the many low probability cases will not "sum to reasonable doubt." To satisfy its burden of persuasion, the prosecution either may directly investigate everyone in Los Angeles, or, more realistically, it may find overwhelmingly incriminating evidence against a single defendant. Both modes of proof are compatible with a Bayesian analysis.

Finally, Ron is correct in observing that "Bayes' theorem . . . explains virtually nothing." The mathematics of probability merely forces a kind of consistency (called "coherence") in the distribution of belief across propositions. The beliefs can be silly or wise, ignorant or knowledgeable. Epistemology and logic are not the same. The "nature of juridical proof" requires an understanding of both.

Date: 8 August 1994

From: Richard.Friedman @um.cc.umich.edu

Peter is right that much of the law of evidence consists of excluding evidence because we are afraid the lawyers will *not* do a good enough job of persuading the jury of the defects in the evidence—or at least that the jury will not satisfactorily perceive those defects. On the other hand often we *DO* rely on the lawyers for just that purpose. So there's a tension here that is resolved case-by-case, or at least issue-by-issue.

14. See David Kaye, *The Laws of Probability and the Law of the Land*, 47 U. CHI. L. REV. 34 (1979).

I guess I'm agnostic as to whether the type of argument to the jury suggested by Roger should be the most a party can do to explain the significance of the evidence to the jury. I'm relieved to know his concern isn't with the basic idea of a prior—which it seems is unavoidable, being simply the evaluation of the probability of the hypothesis made without the evidence in question—but rather with the numerification of both it and the impact of the evidence on it. On the one hand, I'm reluctant to withhold from the jury useful information that would help truth determination if the jury is able to use it; on the other hand, providing that information might be counterproductive if the jury misuses it. So to me, it depends on the empirical question of jury use or misuse, as to which I'd like to see more.

Peter is certainly right about the negation of the hypothesis being a disjunction. For that matter, the hypothesis itself is a disjunction, but that may not be significant. So he's got to be right that the evidence has to be presented in pieces, depending on how significantly different the components of the disjunction are. Thus, DNA evidence might be presented in the following form:

If the specimen at the crime scene were left by the accused, the probability is virtually 1 that it would have the characteristics that I have shown that the specimen does. If it were left by a cousin of his (I don't know that he has any, so I'm assessing the probability based on the information I have from his sample), the probability of a match would be 1 in 50,000. If it were left by a white male chosen at random from the population, the probability is 1 in 1 billion. If it were left by a black male chosen at random from the population, the probability is 1 in 1.5 billion.

It seems to me that giving these individual components is more precise than giving individual likelihood ratios; perhaps, though, sometimes the components would give a false sense of precision.

Clearly, more information could be given, if we know it, by disaggregating the groups, and sometimes that might be significant. But also, in some cases even this much disaggregation might not be necessary, for two or more groups stand in a materially similar posture with respect to the evidence. So I think we naturally batch hypotheses, in a way to make the problem intellectually tractable and the demand for information economically feasible. I wrote some about this batching in *Infinite Strands, Infinitesimally Thin: Storytelling, Bayesianism, Hearsay and Other Evidence*, 14 CARDOZO L. REV. 79 (1992). I think the complexities that Peter points out illustrate a point I made, that "the world is a very complex place, but that is not the fault of Bayesian analysis." When the disjunction of an hypothesis is significant, *any* system of thought that has even a superficially plausible claim to rationality is going to reflect enormous complexity in taking the disjunction into account in a sensible way.

Moreover, I think the same complexities illustrate what I consider to be a problem with the approach to civil trials that Ron has presented, or at least with the form of it that I understand his first writings on the subject to present, comparing *the* story presented by one side against *the* story presented by another. The problem, as suggested by the DNA grouping problem, is that there are an infinite number of stories on either side of the case, each of infinitesimal probability. Making the factual determination depend on the probability of one story against another introduces an artificial intrusion into the factfinding process. (That does not contradict the fact that often a party will choose to emphasize one storyline.)

I think I have one small quibble with the second view of the presumption of innocence presented by Kaye and Balding. I don't think it's enough to take the presumption as being equivalent to the assumption that the accused is no more likely than anyone else in the population to have committed the crime: The jury must begin with the presumption not only that the accused didn't commit the crime but also that no crime was committed. This, I think, makes the matter significantly more complicated. So I think we should probably think about the presumption something like this:

Ladies and gentlemen of the jury, you may not treat the bringing of the accusation, or the prosecutor's opening statement, as evidence that the accused committed the crime charged. Accordingly, now, before the case has begun, when you have no information on the matter apart from your general knowledge of the world, if I were to ask you to assess the probability that the accused is guilty, you must rationally assign a very small probability. That is how you must begin the case.

If we needed to (I'm not sure we would), how could we make such a probability assessment and yet avoid the distortion created by posing the question? Perhaps by burying the propositions of interest in a long list of requests for probability assignments: "How probable do you think it is that on July 8, 1994, a comet hit Mercury? Venus? Earth? Mars? Jupiter? Saturn? That on June 12, Orenthal J. Simpson of Brentwood murdered his former wife, Nicole Brown Simpson, and a waiter friend of hers, Ronald Goldman, at 10:35 p.m.? at 10:36? at 10:37? That on June 12, John Major of Downing Street, London, murdered his wife at 10:35? At 10:36? . . ."

Date: 9 August 1994

From: Alexander Stein <msalex@pluto.mscc.huji.ac.il>

A short comment on David Kaye and David Balding's last point: The mathematics of probability attempt to do a bit more than that, namely: to postulate (1) that beliefs ascribable to different propositions of fact are commensurable; and (2) that they can only be coherent if framed as propositions about chances to capture the reality, rather than, say, proposi-

tions about the strength of evidential support. Bayesian (or other mathematical) probability is a kind of logic, not an exclusive one, and I thought that this is exactly what Kaye says in his *Chicago Law Review* article (which I take as exemplifying a non-imperialistically Bayesian approach). Am I wrong?

Date: 9 August 1994

From: Ron Allen <RJALAF@nuls.law.nwu.edu>

I agree with most of what David Kaye and David Balding have to say, but there remain a few small problems:

1. On my point about 5,000,000 suspects, let's be sure we agree on the hypothetical. There are 5,000,001 people with the opportunity to have killed Nicole Simpson. Lots of evidence points to Simpson. Now, the next piece of evidence is discovered. Common sense suggests it, too, is incriminating of Simpson. But what about Bayes's Theorem? So far as it is concerned, that piece of evidence's implications with respect to Simpson cannot be known without knowing its implications with respect to the remaining 5,000,000. Where does that knowledge come from? And, what would the equation look like that would work out the implications of this evidence, even if we knew how it related to each of the other 5,000,000? It would be pretty long—if each character were the size of this type, I bet the equation would be longer than the observable universe. So, what can we do? Well, we can simply disregard the formalisms of Bayes's Theorem and act as though there really aren't 5,000,000 other possibilities even though there are. That's hardly a powerful argument for this formalism, though. So, what do we actually do? We disregard these kinds of formalisms and apply other forms of reasoning.

2. The consistency forced by Bayes' Theorem is very weak indeed. Suppose a calculation is done. A juridical fact finder is not stuck with the answer. He or she can take the result as evidence that the priors were wrong, readjust them, and reach the "correct" decision. Unless we know what the priors are, that is, truly know them, there is nothing objectionable to this at all. But of course, it does demonstrate again that Bayes's Theorem has virtually no explanatory power at all.

3. Kaye and Balding pick up on Bernard's claims about the uniqueness of conventional probability theory. No mathematician to my knowledge has ever taken that position. In addition, Kaye and Balding claim that all other systems lead to inconsistent belief sets. Inconsistency may not be such a bad thing; rather obviously you need to know what the alternatives are. More to the point, however, I would like this proposition defended (or if indefensible, dropped). Perhaps someone would provide the demonstration of inconsistency that emerges in any inductive logic, Cohen's or others.

Date: 10 August 1994

From: Lewis Henry LaRue <LHL@fs.law.wlu.edu>

You say that the Bayesian analysis demands that one have a calculation for each of the five million others before one can do the calculation for O.J. Why so? I have never read anything by a Bayesian that so asserts, and I do not understand why it follows from the axioms of probability, from which Bayes' Theorem follows.

Date: 11 August 1994

From: Bernard.Robertson@Hertford.oxford.ac.uk

Let us get back to basics:

1. If we had complete knowledge we would know whether chummy did it or not.

2. The only reason we have to make probability assessments is that we have incomplete knowledge.

3. It therefore makes no sense to say that we cannot make a probabilistic assessment in the absence of certain information; the whole point is that probability theory offers the optimum use of the information we do have.

4. It is therefore as sensible to say that we cannot assess a likelihood ratio in the absence of information relating to each other possible perpetrator as to say that we cannot be sure beyond reasonable doubt without also investigating everyone else in the world and eliminating them. There may be a point to what is being said, but there just is not the time and money to do this.

5. Obviously our assessments on any issue might change if we had more information. But what Peter is saying moves in the direction of believing that there is a "true probability" or a "true likelihood ratio." There cannot be any such thing; there are only assessments conditioned on information.

6. Everything above is just as true of ordinary evidence as it is of forensic scientific evidence. This recent exchange illustrates two points:

a. any tendency towards the Mind Projection Fallacy in probabilistic argument opens the door to those who wish to say that probability theory does not apply to their particular kind of process¹⁵ and

b. any argument that Bayesian inference is too complex or impossible in the light of missing information is simply an argument that decision making in general is extremely complex or even impossible. In the absence

15. See Bernard Robertson & G.A. Vignaux, *Probability—The Logic of the Law*, 13 OXFORD J. LEGAL STUD. 457, 460 (1993) (explaining the "Mind Projection Fallacy" as "the fallacy of regarding probability as a property of objects and processes in the real world rather than a measure of our own uncertainty.").

of perfect information the only way to prevent error is to stop making decisions.

Date: 10 August 1994

From: Richard.Friedman@um.cc.umich.edu

On Ron's point about the five million others: If the new evidence has a differential impact on the probability of guilt of the five million others, then any rational system had better, as a theoretical matter, take into account that complexity. (Of course, even if the evidence does distinguish to some slight degree among all the five million, the real human mind cannot handle all that complexity; some batching is inevitable.) And if the new evidence does not discriminate among the five million others, Bayesian analysis takes it into account without any extra problem created by the fact that there are five million, rather than one, alternative hypothesis.

I don't think Ron's reference to "knowing" the prior really makes sense within the framework of subjective Bayesianism, where probability assessments are subjective. But I think he's probably right that one might assess a prior, incorporate new evidence and update the probability assessment according to Bayesian principles (*not* simply Bayes' Theorem), find himself uncomfortable with the posterior probability that results, and re-evaluate the prior. I don't think there's any problem with that—it represents a sort of reflective equilibrium.¹⁶

All right, here's a possible ground of compromise. It seems to me that the idea of probability ought to be reserved for the standard probability calculus, but that assessing the probability of a proposition is only one aspect of a problem. There is also the aspect that we might term "robustness." For instance, we might say in a given case, "Based on what I know, I rate proposition X as 70% probable, but I know that I have very little information on the point, and that with other, relatively accessible information, I might alter my assessment significantly." Example: There are 50 regular icosahedrons (solids each having 20 equilateral triangles for its faces) in a bag, 28 with all red faces, 10 with 14 red faces each, and the remaining 12 with no red faces. If we pick one out of a bag blindfolded, the probability that if we roll it a red face will end down (assuming it will land on one face) is $[(28 \times 20) + (10 \times 14)] / [50 \times 20] = .7$. But we also know that if we open our eyes we might radically alter that assessment, perhaps making it zero, perhaps making it one, and perhaps leaving it at .7. Thus, this is a non-robust assessment.

16. Cf. JOHN RAWLS, A THEORY OF JUSTICE 48 (1971) (using "reflective equilibrium" to refer to an interactive process of adjusting intuitions and general principles in moral reasoning).

Now suppose that we do open our eyes and find that we have picked one of the solids with fourteen red faces. So the probability that red will land down after the roll is .7, as before. But this is a far more robust assessment, because there is no readily accessible information, short of the roll itself, that will cause us to alter this assessment. And in some psychological sense we may be more satisfied making determinations based on this more robust assessment.

I don't know whether this idea of robustness (which I have not worked out much more than the presentation here) corresponds, roughly or not, to any well developed concept in probability theory. But I have a strong suspicion it accounts for much of the continuing fascination with L.J. Cohen's calculus—he is really thinking more about robustness than about probability, I suspect—and for the resistance to accepting verdicts, in blue bus cases and the like,¹⁷ based on propositions that seem probable but that have little or no individuated support. I also think Bayesians should have no problem with this idea of robustness—it expresses something that the individual probability assessments do not. (Bayesians really would be imperialistic if they claimed that all we need to know about a proposition was expressed in the probability assessment.) I think it may also be what Neil Cohen was after several years ago when he tried to invoke in this context the standard concept of confidence levels,¹⁸ and drew a strong response from David Kaye for applying that concept to a realm where it did not fit.¹⁹

Anything to all this?

Date: 10 August 1994

From: Alexander Stein <msalex@pluto.mscc.huji.ac.il>

Jonathan Cohen indeed employs this idea which originated from J.M. Keynes (who used the term "weight" rather than "robustness").²⁰ The Bayesian approach cannot easily accommodate this notion.

17. The blue bus hypothetical appears to have originated in print with Laurence Tribe, *Trial by Mathematics: Precision and Ritual in the Legal Process*, 84 HARV. L. REV. 1329 (1971). For an account of its roots, see James Brook, *The Use of Statistical Evidence of Identification in Civil Litigation: Well-Worn Hypotheticals, Real Cases, and Controversy*, 29 ST. LOUIS U. L.J. 293-352 (1985).

18. Neil Cohen, *Confidence in Probability: Burdens of Persuasion in a World of Imperfect Knowledge*, 60 NYU L. REV. 385 (1986)

19. D.H. Kaye, *Apples and Oranges: Confidence Coefficients Versus the Burden of Persuasion*, 73 CORNELL L. REV. 54 (1987).

20. See L. Jonathan Cohen, *The Role of Evidential Weight in Criminal Proof*, 66 B.U. L. REV. 635 (1986), reprinted in Tillers & Green, *supra* note 2, at 113.

Date: 11 August 1994

From: Bernard.Robertson@Hertford.oxford.ac.uk

Regarding Richard's comments: "robustness" is indeed a standard concept, it is the converse of "sensitivity." But "confidence intervals" are not Bayesian. A likelihood ratio gives a single figure because *all the information is contained in the conditioning*. As long as you are clear about what information you are conditioning on then there can only be one value for a probability assessment. What goes wrong in orthodox statistics is that instead of conditioning on the survey result, they start to condition on the frequency as estimated by the survey. The survey result now becomes an "estimator" and then say that there is uncertainty about how good an estimator it is.

If you have some idea that some fact you have not been able to examine properly will affect the issue in a particular way then you have background information that you should make the best possible use of. For instance, you might tell me that the average maximum temperature in Michigan in November is four degrees. If I have to assess the average temperature in December using that information, I will apply my background expectation that it will get colder. But if you ask me what the average height of a Michigan adult male is given that the average height of a Minnesotan adult male is 5'9", then what am I to do? I have no idea that there is any difference, so I will say 5'9" if I have to reply. Any "confidence interval" I state is just speculation about what might happen if I had information I do not have. Even that is not a correct statement of the definition of confidence intervals, which is so bizarre that no one outside statistics can believe it when they are told it!

Date: 12 August 1994

From: Peter Donnelly <P.J.Donnelly@qmw.ac.uk>

1. Bernard claims that "As long as you are clear about what information you are conditioning on then there can only be one value for a probability assessment." This is false, at least in the world of subjective probability. The rules of probability force coherence amongst an individual's probability assessments. They do not force distinct individuals to have the same subjective probability assessments for particular events (even when both have identical background knowledge).

He also said that "the likelihood ratio gives a single figure." I am not sure exactly what he means by "likelihood ratio" here. If he is referring to the ratio of the probability of a particular piece of evidence given that chummy did it to the probability of that evidence given that some other particular individual (e.g., Richard Friedman) did it (in each case conditional on background information) then the value of this ratio may be different for

different individuals, exactly because their subjective assessments of the two probabilities may differ. Indeed, I have seen actual criminal cases in which different experts assess different values for the likelihood ratio.

Instead, by "likelihood ratio," Bernard may mean (as is more common) the ratio of the probability of that piece of evidence given that chummy did it to the probability given that chummy did not do it (again conditional on background information). The major point of my last message was that for types of evidence (like DNA) whose probability (conditional on the background information) changes as the identity of the assumed criminal changes, the correct Bayesian evaluation of the likelihood demands first an assessment of the likelihood ratio (like that in the previous paragraph) for each possible true criminal, and then a combining of these (as something like a weighted harmonic mean, as it happens) in a way which depends on the (subjective) probabilities (conditional on the background information) that each possible true criminal is actually the true criminal. Here the likelihood ratios being combined may differ with different assessors, as might their "priors," that is their probability assessments of how likely each possible culprit is to be the true criminal, so it is not clear that there is a single figure for the likelihood ratio.

Incidentally, I don't mean this (or my last message) as a criticism of a Bayesian approach. I draw the same conclusion as Richard: the decision framework is complicated and so it isn't surprising that the Bayesian solution is as well—any sensible solution should be. Of course, in practice, one would aggregate collections of possible criminals into a small number of groups (with similar or roughly similar likelihood ratios) before doing the combining. Nonetheless, I find it reassuring to know exactly what the complete Bayesian solution involves, among other things as a yardstick with which to compare other approaches, for example various (over?) simplifications of the Bayesian solution.

Nor am I worried that Bernard's claim that there can only be one value for a probability assessment is false. In the legal context I presume this is consistent with a view that different jurors may interpret differently or give different weight to particular pieces of evidence. It seems more problematical in practice that different experts could legitimately arrive at different likelihood ratios.

2. I agree with Bernard and others that probabilities are assessments on the basis of incomplete information. All of the above details of the recipe for combining likelihood ratios, and for the evaluation of the individual likelihood ratios, rely on this.

My response to the problem of the five million is that we can (must) in principle assess all the relevant probabilities on the basis of the information we currently have. There is no requirement to go out and do more detective work (though this isn't precluded). We know in principle how to allow (in

a Bayesian framework) for the new piece of evidence obtained (the method is described above). In practice this is prohibitive, so we do our best to approximate the correct method (by aggregation and other procedures) and in addition try to judge how good our approximation might be. As I understand it, Ron would have us ignore, rather than approximate, the Bayesian solution.

3. Bernard claims that uncertainty about an estimator (by which, informally, I mean something to do with our confidence in its value, but certainly not a formal confidence interval) is a red herring. Although it is not central to issues of the interpretation of statistical or probabilistic or other evidence, I believe the contrary. The reasoning gives a nice illustration of the consequences of Richard's robustness idea, although I appreciate that he wasn't thinking of its consequences in this context. (Incidentally, while Bayesians do talk about robustness, I think by this they mean the extent to which the posterior is affected by changes in the prior. Richard has in mind the extent to which it may be affected by additional information.)

Suppose we have a new form of identification evidence—some trait that may or may not be present in an individual. Our assessment of the probability that a member of some relevant population will possess the trait is 0.5. Suppose this assessment depends in whole or in part on a sample survey of that population. We know that the criminal and our particular defendant possess this trait, and intend in good Bayesian fashion to calculate the likelihood ratio. One convenient method of evaluating both numerator and denominator involves conditioning on the fact that the defendant has the trait. We will do it this way, but the conclusion doesn't depend on this method of evaluation. In the denominator we need to assess the probability that some particular individual (distinct from the defendant) possesses the trait, given that the defendant does. (We will assume for simplicity that unlike DNA this conditional probability is the same for each other individual in the population.) As a consequence of the conditioning, the background information has changed. We now know the status of a particular individual, the defendant, and in particular that this individual has the trait. This change in the background information will in general change our assessment of the probability that an untested individual from the population will have the trait. (In general this particular new information should increase the probability, or at least not decrease it.) For example, if our original assessment of $\frac{1}{2}$ was based on a survey of four individuals, two of whom had the trait, our new assessment may be close to $\frac{3}{5}$. If, on the other hand, the original probability was based on an exhaustive survey of the population (from which we knew the number of trait bearers but not their identities) it may not change at all. The point is that the probability we assess in calculating the likelihood ratio (and hence the likelihood ratio itself) will depend not only on the population frequency from the survey, but also on our uncertainty about that

frequency, or put in a way Richard may have chosen, it depends not just on our first order assessment of the probability (on the basis of the background information but not the trait evidence) but on the (second order) uncertainty about it.

Date: 11 August 1994

From: DH Kaye <k@asu.edu>

Of the various observations that David Balding and I offered concerning *Skipper* and some of the correspondence about it, two have prompted responses. Further thoughts on these, as well as some references to work by Bayesians and others on second order probabilities and related concepts to cope with what Rich calls "robustness," are presented below. These remarks were written before the latest missives from Peter Donnelly and Bernard Robertson.

*1. Evidence of One Person's Involvement is Evidence
Against Everyone Else's Involvement*

We don't need the details of Bayes' rule to appreciate that the total probability "mass" sums to one and is distributed in varying amounts over the 5 million and one suspects. Ron posits new evidence that "common sense suggests is incriminating of Simpson." If so, more probability must be assigned to the hypothesis of Simpson's guilt, and this must come from the probability mass attached to the other five million suspects. This much follows directly from the axioms for probabilities, as Lash notes. Beyond requiring that what goes to Simpson comes from the other five million, the axioms do not constrain how much probability mass to remove from every other individual. Coherence is a weak constraint, and it accommodates a "common sense" understanding of how much the probability for Simpson increases.

One can get to this result with Bayes' rule, of course. Let i denote each suspect ($i=0, \dots, N$), where $N = 5$ million. Let 0 denote Simpson, and let S_i stand for the proposition "suspect i is the killer." Before Ron's new evidence (E), the probability of S_0 is some number p_0 . To say that E incriminates S_0 means that the likelihood of S_0 is greater than the likelihood of S_i for all $i>0$. That is, $\lambda(S_0 | E) > \lambda(S_i | E)$, where $i=1, \dots, N$. According to Bayes's rule, we find the new probability for S_0 by multiplying p_0 by $\lambda(S_0 | E)$ and dividing this product by the sum of each prior probability (from $i=0$ to N) weighted by its likelihood:

$$p(S_0 | E) = [p_0 \lambda(S_0 | E)] / \sum p_i \lambda(S_i | E). \quad (1)$$

This equation is not "longer than the observable universe."

Ron's objection may be that we don't have the likelihoods with which to compute (1). "Where does that knowledge [of the implications of the evidence] come from?", he asks. In the absence of a concrete example, I suppose it must come from whatever was the source of the revelation that E incriminates S_0 . Richard suggests that someone who wanted to fuss with (1) could group various suspects into categories and come up some manageable set of values for the likelihoods. I suspect that someone like Ward Edwards, who has thought hard about decision analysis,²¹ would be able to handle any concrete example that Ron could devise. Computational complexity may counsel against using Bayes's rule as an explicit tool at trials, but it is not a strong objection to using probability theory as an analytical tool. The proof of the value of this approach lies in the pudding, and the "Michigan School" has produced some treats in this kitchen.²²

2. Consistency and Probability

Ron's comments about mathematical demonstrations reminds me that a few years ago, I tried to popularize what I thought was a well known result among students of inductive logic and the foundations of probability. I wrote that:

Another argument builds on the same idea of assigning plausibilities to all propositions p , q , r , and so on, as well as more complex propositions such as $p \& q$, $p \text{ or } q$, and $\neg(p \& q)$. It can be proved that the only way to assign the same plausibility to all logically equivalent propositions is to use plausibilities that obey the rules for probabilities. In addition, the only way to maintain logical consistency when a new proposition z is added to our stock of knowledge is to readjust all the plausibilities by conditioning on z , that is, by changing $Pl(p)$, $Pl(q)$, $Pl(r)$, ... , to $Pl(p | z)$, $Pl(q | z)$, $Pl(r | z)$, ... , where $Pl(p | z) = Pl(p \& z) / Pl(z)$, This rule of conditionalization is just Bayes's rule.²³

Many of the proofs in the literature are cast in terms of betting odds, but "the possibility of a Dutch book [is] a symptom of a deeper pathology. The bettor who violates the laws of the probability calculus leaves himself open to having book made against him because he will consider two different sets of odds as fair for an option depending on how that option is described,

21. See, e.g., DETLOF VON WINTERFELDT & WARD EDWARDS, *DECISION ANALYSIS AND BEHAVIORAL RESEARCH* (1986).

22. See, e.g., Richard Lempert, *The New Evidence Scholarship: Analyzing the Process of Proof*, 66 B.U. L. REV. 439 (1986), reprinted in Tillers & Green, *supra* note 2, at 61; Richard D. Friedman, *Character Impeachment Evidence: Psycho-Bayesian [!?] Analysis and a Proposed Overhaul*, 38 UCLA L. REV. 637 (1991); Richard D. Friedman, *A Close Look at Probative Value*, 66 B.U. L. REV. 733 (1986).

23. D.H. Kaye, *Credal Probability*, 13 CARDOZO L. REV. 647, 656 (1991), citing BRIAN SKYRMS, *CHOICE AND CHANCE: AN INTRODUCTION TO INDUCTIVE LOGIC* 193 (3d ed. 1986).

the equivalence of the descriptions following from the underlying Boolean logic.”²⁴

I hope this clarifies the claim that David Balding and I made about the appeal of probability theory as an inductive logic. The proofs do assume, as I observed in the *Cardozo* comment, that plausibilities must be assigned to all propositions; so, one can evade the result by refusing to grade the probable truth of some propositions. Alex Stein makes a related point when he speaks of “beliefs ascribable to different propositions of fact [being] commensurable.” Although Cohen’s ordinal “probabilities” also seem to presuppose “commensurability,” I am not prepared to say that they suffer from inconsistencies. Still other approaches, such as upper and lower probabilities, avoid incoherence by not grading probable truth in terms of a single-valued function. In sum, I must concede that the situation is more complicated than the Kaye-Balding note suggested. The status of the “representation theorems” and “Dutch book” arguments for probability are thoughtfully assessed in, e.g., PATRICK MAHER, *BETTING ON THEORIES* (1993).

As for the putative weakness of Bayesian inference because we do not “truly know” the prior distribution (point 2 in Ron’s note of August 9), I want to reinforce Rich’s August 10 remarks about “reflexive equilibrium.” Ron acts as if the decision maker is free to pick new prior distributions. Formally, there is such freedom. Bayes’s rule itself does not prevent a decision maker from revisiting the question, but a decision maker who takes Bayesian decision theory seriously will not ordinarily regard prior assessments as arbitrary. The individual who must decide may come to believe that the prior distribution is mistaken, but he or she will not cast a distribution aside whimsically. Indeed, I believe that this kind of result can be established within a Bayesian framework that considers doing a calculation to determine what act maximizes expected utility as itself an act. If one approaches the task of setting and revising priors capriciously, the use of the calculation may not maximize expected utility.

The theory of how beliefs should change over time often goes under the heading of “probability kinematics.”²⁵ As far as I know, this rich literature is not reflected in the writings of legal scholars.

Finally, I am inclined to disagree with Alex on one matter. The school of logical probability (of which Keynes is often cited as a founder), as I understand it, uses probabilities to grade “the strength of evidential support” for a proposition. In this interpretation of the probability calculus, a

24. Brian Skyrms, *Dynamic Coherence*, in FOUNDATIONS OF STATISTICAL INFERENCE 233-243 (J.B. MacNeill & G.J. Umphreys eds., 1987).

25. See, e.g., Persi Diaconis & Sandy L. Zabell, *Updating Subjective Probability*, 77 J. AM. STAT. ASS’N 822 (1982); Brian Skyrms, *Dynamic Coherence and Probability Kinematics*, 54 PHIL. SCI. 1 (1987).

conditional probability gives the strength of an inductive argument. It expresses the degree to which a conclusion follows from other propositions. Again, I do not want to say that the probability calculus is the only way to approach problems of inductive proof and confirmation, but the logical relation theory of probability springs from this tradition.

3. Imprecise Probabilities and Weight

Rich asks (August 10) whether the “idea of robustness” of probability judgments is a “well developed idea in probability theory.” Again, there is considerable writing about the idea, which goes by other names. As Alex observed, Keynes mentioned “weight” as distinct from probability, but Keynes questioned whether the idea had any importance or practical significance.²⁶ It appears that the question of the completeness of the evidence can be handled within the usual Bayesian framework, but one also can account for the completeness of the evidence with a “second-order” probability.²⁷

Date: 13 August 1994

From: Bernard.Robertson@Hertford.oxford.ac.uk

I have not been using words like “subjective” and “objective” and it seems to me that they are not useful labels. All probability assessments are personal in the sense that we each have a unique set of experiences and background information to condition upon. But all probability assessments are also “objective” in the sense that they are based upon a rational

26. See D.H. Kaye, *Do We Need a Calculus of Weight to Understand Proof Beyond a Reasonable Doubt?* 66 B.U. L. REV. 657, 657 n.3 (1986).

27. Second-order probability might be capable of capturing the notion of subjective confidence that Professor Neil Cohen wrongly ascribes to a frequentist confidence interval. Cohen, *supra* note 16. In D.H. Kaye, *Do We Need a Calculus of Weight to Understand Proof Beyond a Reasonable Doubt?* 66 B.U. L. REV. 657, 667-68 n.22 (1986), reprinted in Tillers & Green, *supra* note 2, I declined to pursue this possibility, however, “in part because I lack the technical expertise, and in part because the first-order model that I have sketched should give the same results. See Brian Skyrms, *Higher Order Degrees of Belief*, in PROSPECTS FOR PRAGMATISM 109, 167-68 (D. Mellor ed., 1980) (“first-order models are good medicine for acrophobia”). Further citations to the technical literature may be found at D.H. Kaye, *Apples and Oranges: Confidence Coefficients Versus the Burden of Persuasion*, 73 CORNELL L. REV. 54, 77 (1987). See also COLIN HOWSON & PETER URBACH, SCIENTIFIC REASONING: THE BAYESIAN APPROACH 68-70 (1989) (probability intervals); Nils-Eric Sahlin, *On Second Order Probabilities and the Notion of Epistemic Risk*, in FOUNDATIONS OF UTILITY AND RISK THEORY WITH APPLICATIONS 95-104 (B.P. Stigum & F. Wenstop eds., 1983); BRIAN SKYRMS, CAUSAL NECESSITY: A PRAGMATIC INVESTIGATION OF THE NECESSITY OF LAWS 12-13 (1980) (defining the “resiliency” of a conditional probability). For empirical work, see, e.g., Robert W. Goldsmith & Nils-Eric Sahlin, *The Role of Second-Order Probabilities in Decision Making*, in ANALYZING AND AIDING DECISION PROCESSES 455-467 (P.C. Humphreys et al. eds., 1982).

assessment of the available information. I do not see how two individuals given identical information can come up with different assessments. If they do, they must each be able to justify their assessment rationally by reference to their information in which case the reason for the difference will become plain and one or both will wish to adjust their assessments.

There cannot be any room for saying "Well, I feel that this is the correct value." Sometimes intuitions are in fact the subconscious amalgamation of a huge quantity of information of which, as I.J. Good says, "much is half-forgotten." But to say that one somehow has a "right" to make one's own assessments which derive their validity purely from the fact that they are mine and "subjective" assessments are allowed, is to descend to cultural relativism.

The reason, of course, that in practice people make different assessments is that we are each unique with a unique stock of experience. A multi-member jury is an attempt to iron these differences out.

Further elucidation: A perfectly good reason for aggregating is that there is no point in trying to distinguish between items we have no information enabling us to distinguish between. Consider a murder in Chicago. To complete our information we might want a prior distribution for each inhabitant of Mongolia (some have access to foreign travel, some don't, etc.). But we don't have this information, and we don't care to gather it, so we just regard the inhabitants of Mongolia as an aggregate, until we discover that the victim is a Mongolian dissident.

In this connection I have tried to question some of the abuse which has been heaped upon *R. v. Blastland* [1986] 1 AC 41. In that case the House of Lords say that evidence which tended to increase the probability that A committed an offence was not relevant (and therefore not admissible) to the question of whether B committed it. Now without judging the facts of that particular case, it seems to me that this is not necessarily wrong in principle as many critics seems to assume it is. In the example of a murder in Chicago, evidence that a particular Mongolian visited the U.S. during this period must increase one's assessment of the probability that he was the perpetrator, but not necessarily at the expense of the probability assessment of the victim's next-door neighbour; rather it enables one to redistribute the probability assigned to all inhabitants of Mongolia.

The evidence in *Blastland* indicated that the second person was in the vicinity at the time. But so might lots of other people have been so any adjustments may take place only within the $1 - P(\text{Accused's guilt} \mid I)$.

Date: 13 August 1994

From: Ron Allen <RJALAF@nuls.law.nwu.edu>

A few general words. My view is not that any of the formalisms we are arguing about are wrong in any serious sense. The question I am trying to

examine is the nature of juridical proof. For that purpose, I find the various formalisms useful tools, but none of them is the bedrock explanation. On a normative plane, to which I rarely venture I must confess, I don't find any of them particularly attractive as singularly important method of proceeding, although all of them together are enormously useful. Human rationality has numerous attributes and has created numerous tools to assist it in its efforts. The intriguing thing for me in this e-mail debate is the reductionist claims of some Bayesians, that's all. I have no doubt of the occasional utility of statistical methods. So, quite to the contrary of Peter's suggestion that I would disregard Bayesianism altogether, I'd allow it as evidence when useful, so long as it was properly employed and properly explained. For those interested (if any) in the details of these views, they are pretty much laid out in *Factual Ambiguity and a Theory of Evidence*, 88 NW. L. REV. 604 (1994), and *The Common Law Theory of Experts*, 87 NW. L. REV. 1131 (1993). . . .

Date: 22 August 1994

From: Richard.Friedman@um.cc.umich.edu

Under the standard view of the burden of persuasion,²⁸ the burden depends on the disutility of the possible erroneous results (and, to be perfectly accurate, the utility of the possible accurate results). Given this view—which I think captures much, if not absolutely all of what the burden of persuasion is about—I don't see how we could avoid stating the burden of persuasion in standard probabilistic terms. Probability is, after all, defined in these very terms—that is, decision making under uncertainty given various gains and losses for correct and incorrect decisions.

But this does not mean that the probability that would justify a finding for the plaintiff in a civil case is always the same. Usually we talk in terms of "more likely than not" (though I suspect we mean "substantially more likely than not"), but in some types of cases (as where the state seeks to commit someone involuntarily to a mental institution) we demand that the plaintiff's case be "clear and convincing." And sometimes a particular fact about a case may alter the burden of persuasion. For example, in a paternity case the fact of marriage creates a presumption that the child was the legitimate child of the husband and wife, and this presumption not only requires that the party contending otherwise produce evidence in support of the contention, but also imposes on that party the burden of satisfying the fact finder by the "clear and convincing" standard. It is easy enough to understand why: The fact of the marriage is thought to alter the stakes, making an incorrect decision that the husband is not the father much worse,

28. Richard Lempert, *Modeling Relevance*, 75 MICH. L. REV. 1021 (1977).

an incorrect decision that the husband was the father not as bad, and so forth.

So what I am suggesting is simply that the completeness of the evidence (weight, resilience, robustness, quality, whatever) may similarly be a factor affecting the burden of persuasion in a particular case. Why? Because even given the same probability of liability, we have a much less satisfied feeling (at least many people do, and that might be enough) granting relief in a case in which the evidence is very incomplete, even if justifiably so. There may be at work here a feeling of inertia, that because of an “above all do no harm” orientation we should hesitate to grant relief to the plaintiff unless we have a relatively filled-in picture of what happened. So we might say that the less complete the evidence the greater the probability must be to warrant relief for the plaintiff. In other words, instead of a very simple rule of the form $P > K$, where P is the probability of liability and K is the designated standard of persuasion (.5, generally), we might have a rule that includes two factors. Perhaps, if we could formalize it (which would probably be difficult and I don’t think is necessary), we could state such a rule in the form $PQ > K$, where Q is some measure of the quality or weight of the evidence and K represents the standard that the combination of the two factors must meet to justify a verdict for the plaintiff. Or, if we prefer stating the rule by identifying a probability that the plaintiff must surpass, $P > K/Q$.

In my view, this analysis does not require any alternative notion of probability, or any mode of reasoning inconsistent with standard probabilistic reasoning. P and Q may be “incommensurable,” but only because they represent different concepts, in the same way that mass and time are incommensurable. This analysis recognizes the critical role of probability in understanding burdens of persuasion, but it also recognizes that, like most legal rules that do not operate as simple cut-off points on a (one-dimensional) line, the rule on burden of persuasion *may* take into account factors other than probability—or, put another way, the probability that the plaintiff must exceed depends on the facts of the case—and one of those factors may be the completeness of the evidence. In other words, it seems to me there is ample room under the “Pascalian” tent for whatever useful measures of weight that Baconians may have to offer.

One final thought. The continuing resistance to the idea that Bayesian analysis can be useful in studying matters of juridical proof and evidence seems to me ironic given that some of us have been using that analysis for years and generating results that (in my self-interested view) are not obvious but that are sound, intuitively appealing, and readily explainable.